6.4 exponential growth & decay

Suppose you deposit \$800 in an account that pays 6.3% interest. How much will you have 8 years later if the interest is compounded:

annual interest rate

a) annually $800(1+.063)^8 = 1304.23$

b) quarterly $800\left(1+\frac{.063}{4}\right)^{8\cdot 4} = 1319.07$

c) monthly $800\left(1+\frac{.063}{12}\right)^{8\cdot 12} = 1322.52$

d) daily $800\left(1+\frac{.063}{365}\right)^{8\cdot 365} = 1324.21$

e) continuously ? $800e^{.063\cdot 8} = 1324.26$

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differential equation for continuously compounded interest

$\frac{dy}{dt} = ky$

solve the d.e. for y

1. sep. of var

$\int \frac{dy}{y} = \int k\, dt$

$e^{\ln|y|} = e^{(kt+c)}$

$y = e^{kt} \cdot e^{c}$

y = \$ amount

y = FV

amount after time t

k = growth constant

$y = e^{kt} \cdot c_1$

$t=0 \quad y = 800$

$800 = e^0 \cdot c$

$c = 800$

$y = 800e^{kt}$

If $k = .063$ $y = 800e^{.063t}$

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$$\lim_{t\to\infty}\left(1+\frac{1}{k}\right)^{k} = e$$

compound interest

$$y = y_0\left(1+\frac{r}{k}\right)^{kt}$$

y_0 = starting amount

k = number of times compounded per yr

r = interest rate (decimal)

t = years

continuously

$$y = y_0 e^{rt}$$

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~~10,000~~ 30,000 is invested at 7%. How long to double?

(compounded continuously)

$$\frac{60000}{30000} = \frac{30{,}000\,e^{.07t}}{30000}$$

$$2 = e^{.07t}$$

$$\ln 2 = \ln e^{.07t}$$

$$\ln 2 = .07t$$

$$\frac{\ln 2}{.07} = t$$

$$\frac{2y_0}{y_0} = \frac{y_0 e^{rt}}{y_0}$$

$$2 = e^{rt}$$

$$\ln 2 = \ln e^{rt} = rt$$

double time $t = \frac{\ln 2}{r}$

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$\frac{dy}{dt} = ky$

↑ how fast the amount grows

↑ amount

as y gets bigger,

y ~~↑~~ grows faster

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use continuous growth to model a bacteria coloney

if the coloney doubles in 16 hrs, and there are 700 present initially, what will the pop. be in 52 hrs?

double time $t = \frac{\ln 2}{r}$

$16 = \frac{\ln 2}{r}$ $\quad r = \frac{\ln 2}{16} \approx .043$

$y = 700e^{.043 \cdot 52} = 6549$

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exp. decay (decreasing)

$\frac{dy}{dt} = -ky$

$\int \frac{dy}{y} = \int -k \, dt$

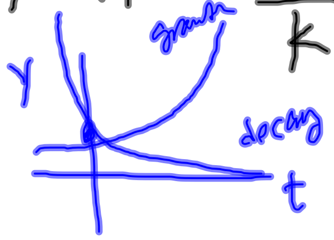
$\ln|y| = -kt + c$

$y = e^{-kt+c}$

$y = e^{-kt} \cdot e^{c}$

$y = c e^{-kt}$

half life $= \frac{\ln 2}{k}$



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